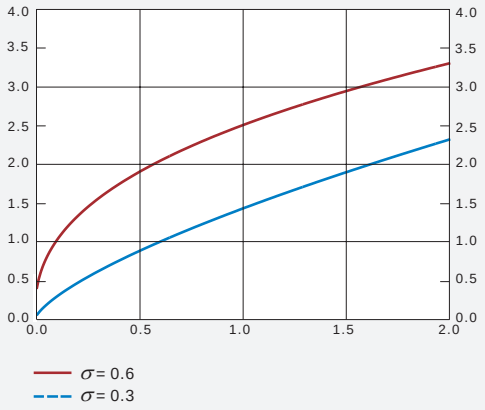
Figure B15. Graphic illustration of the utility function when $\sigma = 0.3$ or 0.6.



Note. The level of consumption, c_t, is plotted on the *x*-axis and the level of utility, $U(c_t)$, on the *y*-axis.

Source: The Riksbank.

THE CONCEPT OF A REAL LONG-TERM EQUILIBRIUM INTEREST RATE

Empirical estimations of real equilibrium interest rates in different countries and periods were presented in *Inflation Report 2000:1* (in a box on pp. 24–28). They suggested that the real equilibrium interest rate can fluctuate a good deal over time and also differ fairly appreciably across countries.

Factors that according to economic theory explain the real equilibrium interest rate are discussed here. Rather than providing estimations of the path or level of this rate, the main aim is accordingly to provide a simple theoretical framework for continued discussions and analyses.

The (long-term) level of the real equilibrium interest rate is determined by households' consumption and saving decisions over time. A simple yet generally applicable approach that is often used to study consumption and saving is the Ramsey model.[33] The utility of aggregate household consumption, $U(c_t)$, is described there by a function of the following form:

$$U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma},$$

where c_t is aggregate household consumption and σ a parameter that is greater than zero. The assumption that s exceeds zero implies that the marginal utility of consumption diminishes, that is, the utility of an increment to consumption declines as the level of consumption rises. This is illustrated in Fig. B15 for two values of σ: 0.3 and 0.6. A lower value of s means that the marginal utility declines more slowly.

The parameter σ is important for several reasons. Besides representing households' attitude to uncertainty, it measures their propensity to substitute consumption between time periods. A high σ means that households dislike uncertainty about the future potential for

33 For a detailed description of the model, see e.g. Campbell, J. (1996), Consumption and the stock market, *Swedish Economic Policy Review*, 3. See also the original paper: Ramsey, F. (1928), A mathematical theory of saving, *Economic Journal*, 38. The model used here does not, of course, include every conceivable factor that can affect the real interest rate. Tax effects, for example, are disregarded; the inclusion of a tax on capital income, for instance, would tend to push the real interest rate up in equilibrium.

consumption, that is, they are averse to unforeseen variations in consumption. It also implies a low substitution propensity. The use of a single parameter to determine both the attitude to uncertainty and the substitution propensity is a consequence of the particular type of utility function that is assumed to apply. There are other types of utility function that enable these properties to be separated, which can sometimes be desirable.

Households have a subjective discount rate or time preference, δ, that represents how they value future consumption compared with consumption today. Their discounting of future consumption can be described as follows:

$$U(c_t) + \frac{1}{1+\delta} U(c_{t+1}) + \frac{1}{(1+\delta)^2} U(c_{t+2}) + \ldots$$

A value of δ that is greater than zero means that households are 'impatient' and value consumption today more highly than future consumption. If δ is less than zero, households are 'patient' and see future consumption as more valuable.

Moreover, households are assumed to maximise their expected utility, subject to the budget restriction, which includes the possibility of investing in an asset that yields a no-risk real return. From this model one can derive the following expression for the real long-term equilibrium interest rate:

$$r = \delta + \sigma\gamma_c - \frac{\sigma^2}{2} Var(\gamma_c),$$

where γ_c denotes consumption's growth rate and $Var(\gamma_c)$ the variance of this growth rate in equilibrium.[34]

The expression states that the real long-term equilibrium interest rate is determined by three factors. One is a simple *time preference* effect. In the case where households are 'impatient' and value consumption today more highly than future consumption, compensation is required to induce them to save and postpone consumption. This compensation consists of interest on their savings.

Another factor is an effect that is dependent on *economic growth*. An expanding economy generates a potential for future consumption that is larger than today's. As the marginal utility of consumption declines

34 This expression is an approximation of the solution in the paper by John Campbell (see previous footnote).

over time, households value increased consumption today more highly than a future increase. They will therefore be prepared to pay a premium or interest in order to be able to increase their current consumption. The interest rate they are prepared to pay will depend on their substitution propensity; a low propensity means they are willing to pay a high rate of interest in order to achieve increased consumption today.

Thirdly, the real interest rate is influenced by *uncertainty about the potential for future consumption*. If this potential is uncertain, households will save in order to have a buffer against a poor outcome. Increased saving tends to lower the real interest rate. The magnitude of this effect depends on the level of uncertainty and households' attitude to uncertainty. The more households dislike uncertainty, the higher will saving be and the lower the real interest rate.

The terms δ and σ represent what are known as preference parameters and arriving at a definite opinion of their value is difficult. However, d is usually assumed to be greater than zero, although there are arguments for setting it close to zero. In his 1928 paper, for example, Ramsey assumed that δ should be equal to zero on the grounds that it would be 'ethically indefensible' to value the utility of future generations at less than that of today's generation.

The value of σ is particularly important because it determines households' attitude to uncertainty as well as their substitution propensity. Studies at household level have found that households are relatively willing to substitute[35] but this behaviour has been more difficult to confirm with aggregated data.[36] Studies attempting to measure households' attitude to uncertainty have found that an upper limit for what can be regarded as reasonable is $\sigma = 4$.[37]

The sensitivity of the real interest rate to some alternative values of δ and σ, with the growth rate and variance given, is shown in Table B3.[38] The level of the interest rate is relatively sensitive to variations in both the time preference and the substitution propensity. The quantitative effect of uncertainty, on the other hand, is small. If, for example, the attitude to

35 See Runkle, D. (1988), Liquidity constraints and the permanent income hypothesis: evidence from panel data (manuscript), Federal Reserve Bank of Minneapolis.

36 See Hall, R. (1989), Consumption, in Barro, R., *Modern Business Cycle Theory*, Harvard University Press.

37 See Romer, D. (1996), *Advanced Macroeconomics*, McGraw-Hill.

38 In the period 1970:1–1995:1 the annual growth rate for Swedish consumption was 0.70 per cent and the variance 0.00037, see Campbell, J. (1999), Asset prices, consumption and the business cycle, in Taylor J. & Woodford, M., *Handbook of Macroeconomics*, Elsevier Science B.V.

uncertainty, σ, is given the value 2, the effect on the real interest rate amounts to only around 0.07 per cent. This may seem remarkable. It could, indeed, well be the case that households' attitude to uncertainty is considerably higher. Another, perhaps more probable, explanation is that uncertainty is higher at individual level than at the aggregated level.

Table B3. Real long-term equilibrium interest rate with alternative values of δ and σ.
Per cent

	σ=0.5	σ=1	σ=2	σ=4
δ=0	0.3	0.7	1.3	2.5
δ=0.02	2.3	2.7	3.3	4.5
δ=0.04	4.3	4.7	5.3	6.5

Source: The Riksbank

In certain contexts the real long-term rate of economic growth is commonly used as an approximation of what the real long-term equilibrium interest rate should be. This accords with economy theory only under certain specific assumptions, however. In the model presented here, it is valid only if uncertainty is disregarded and assuming that δ=0 and σ=1.

Table B4 presents the average real interest rate *ex post* and the growth of consumption for a number of countries since the 1970s. The data for Sweden are also given separately for each decade. The figures for the real interest rate should be interpreted with caution, however, partly because capital markets in Sweden and many other countries were regulated up to the mid 1980s.

Table B4. Real long-term equilibrium interest rate and consumption growth since the 1970s.
Per cent

Country	Period	r	γ_c
Australia	1970 Q1–1996 Q2	1.8	1.8
Canada	1970 Q1–1996 Q2	2.7	1.9
France	1973 Q2–1996 Q2	2.7	1.6
Germany	1978 Q4–1996 Q2	3.3	1.6
Italy	1971 Q2–1995 Q2	2.1	2.4
Japan	1970 Q2–1996 Q2	1.5	3.4
Netherlands	1977 Q1–1996 Q1	3.7	1.5
Sweden	1970 Q1–1994 Q4	1.5	0.8
United Kingdom	1970 Q1–1996 Q2	1.1	2.0
United States	1970 Q1–1996 Q3	1.4	1.7
Sweden	1970 Q1–1979 Q4	-2.0	1.6
Sweden	1980 Q1–1989 Q4	3.7	1.1
Sweden	1990 Q1–1999 Q2	4.6	0.2

Sources: John Campbell (1999), Asset Prices, Consumption and the Business Cycle, John Taylor and Michael Woodford, *Handbook of Macroeconomics,* Elsevier Science B.V., and the Riksbank.

From Table B4 it will be seen that, as an average, the real interest rate has been at approximately the same level as the growth of consumption in most of these countries. At the same time, for some countries the difference between the interest rate and consumption is considerable.

The breakdown into decades for Sweden shows, moreover, that consumption growth has varied considerably over time. This suggests that the long-term growth rate is not constant over time, which in turn may have meant that the long-term equilibrium interest rate also fluctuated over time (see also *Inflation Report 2000*, box on pp. 24–28).[39]

The purpose of this presentation has been to put forward a simple theoretical framework for discussions about the real long-term interest rate. The factors discussed have been households' time preference (their valuation of consumption today relative to future consumption), the long-term rate of economic growth and uncertainty about the potential for future consumption.

In certain contexts the real long-term rate of economic growth is commonly used as an approximation of what the real long-term equilibrium interest rate should be. It is only under certain specific assumptions, however, that economic theory regards this as a good approximation.

39 With today's economic policy commitment to stability in Sweden, however, growth — and thereby the real interest rate — may fluctuate less in the future.